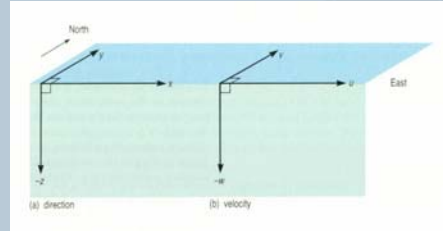


# Continuity

MSCI 301  
Physical Oceanography  
Coastal Carolina University

# Coordinate System

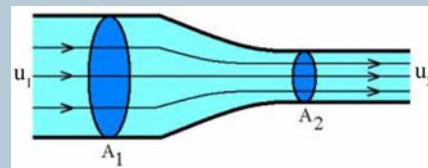
Used for all calculations from now on....



# Continuity

Also known as **Conservation of Mass**

- Mass in = Mass out
- Assumes the fluid is incompressible



Flow Rate

$$\frac{\text{mass in}}{\text{time}} = \frac{\text{mass out}}{\text{time}}$$

$$\frac{\rho_1 V_1}{t} = \frac{\rho_2 V_2}{t}$$

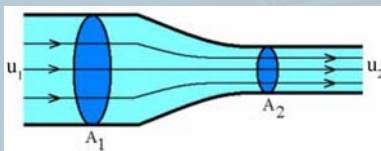
$$\rho A_1 u_1 = \rho A_2 u_2$$

# Flow Rate Problem

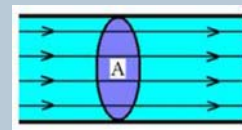
For the pipe below,  $A_1=0.05\text{m}^2$ ,  $A_2=0.015\text{m}^2$ , and  $u_1=0.5\text{m/s}$ . The pipe is 2m long between  $A_1$  and  $A_2$ .

Assuming constant density and an incompressible fluid, what is the velocity of the fluid at  $A_2$ ?

**What is  $du/dx$  ?**



An even simpler problem:



What is  $du/dx$  here?

$$\frac{du}{dx} = 0$$

### 1-D Continuity Equation

$$\frac{du}{dx} = 0$$

- Assuming:

- Incompressible Fluid
- $A_1 = A_2$
- Constant density

- What does it mean?

- No change in velocity from one end of the pipe to the other, so mass is conserved.
- No leaks!

$$\frac{\Delta u}{\Delta x} = \frac{u_2 - u_1}{x_2 - x_1}$$

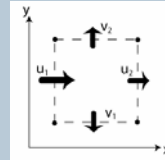
or

$$\frac{\Delta u}{\Delta x} = \frac{u_1 - u_2}{x_1 - x_2}$$



### 2-D Continuity

$$\frac{du}{dx} + \frac{dv}{dy} = 0$$



All sides of the box are 10 m in length.

If  $u_1=5\text{m/s}$  and  $u_2=2\text{m/s}$ , what is  $du/dx$ ?

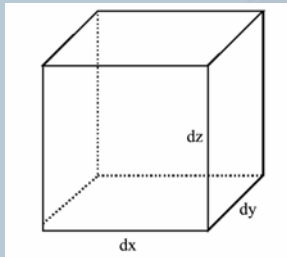
What is  $dv/dy$ ?

What are some ways to get that?



### 3-D Continuity

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0$$

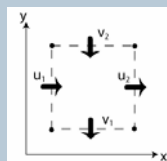


### What's this good for?

- Models of flow in estuaries (we'll do that later in the course)
- Inferring vertical velocities
  - Usually very small



### Continuity Problem:



Each horizontal side of this box is 200 km.

You are on a research cruise in the middle of the ocean. It's your job to determine the vertical velocity of the ocean at the bottom of the mixed layer, which is 50m deep where you are. To do this, you measure the horizontal currents at the four locations marked on the figure. You find that: the speeds are:

$$u_1=0.25\text{m/s}$$

$$u_2=0.35\text{m/s}$$

$$v_1=0.20\text{m/s}$$

$$v_2=0.40\text{m/s}$$

Calculate  $w$  at a depth of 50m.

