

## Vorticity

MSCI 301  
Physical Oceanography  
Coastal Carolina University

## Physics review

- Angular velocity
- Torque
- Angular momentum

## Vorticity

- Vorticity is the rate of rotation of a fluid.
- Two types of rotation
  - Relative (spinning on its own axis)

$$\zeta = \nabla \times \mathbf{V} = \text{curl } \mathbf{V}$$

$$\zeta = \frac{dv}{dx} - \frac{du}{dy}$$

What does this mean?

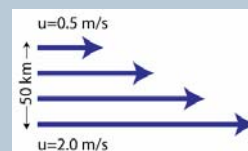
## Relative Vorticity Calculation

A cold core eddy formed off the coast of the US is 150 km wide with a speed at the north end of 1.2 m/s, the south end 1.1 m/s, the west side 1.1m/s and the east side 1.2m/s. What is the relative vorticity of this eddy?

## Sign (+/-) of Vorticity

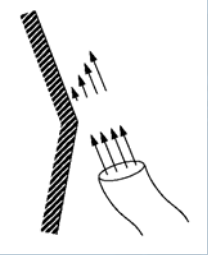
- Counter-clockwise = positive
- Clockwise = negative
- Just like angular velocity

## Does this have relative vorticity?



$$\zeta = \frac{dv}{dx} - \frac{du}{dy}$$

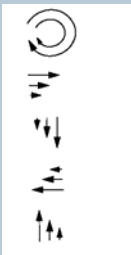
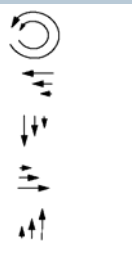
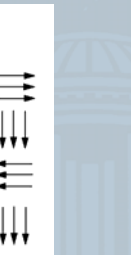
### Friction causing shear



(Najjar)

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### Different ways to get Relative Vorticity

Negative	Positive	None
		

(Najjar)

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### Planetary Vorticity

(rotating because you're on a rotating planet)

$$f = 2\Omega \sin(\phi)$$

Why not just  $\Omega$ ?

What's the biggest that  $f$  can be?

$$\zeta = \frac{dv}{dx} - \frac{du}{dy}$$

- Planetary is usually much bigger

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### Conservation

- What does conservation mean in physics?
- The total amount of something does not change
  - Total energy
  - Angular momentum
  - Mass

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### Conservation of Vorticity

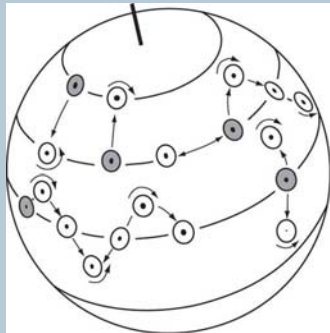
- Absolute amount of vorticity is conserved

$$\text{Absolute Vorticity} \equiv f + \zeta$$

Individual numbers can go up and down, but the *sum* is constant

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Angular momentum tends to be conserved as columns of water change latitude. This causes changes in relative vorticity of the columns



(von Arx)

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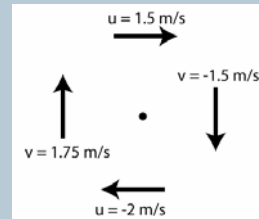
### Vorticity questions

Water parcels move around the ocean, their spin or rotation changes to conserve angular momentum. How will the spin of each of the following eddies change - will they spin faster or slower? Hint: write down the signs of the relative and planetary vorticities, and remember that the absolute vorticity is conserved.

- A) An anti-cyclonic eddy in the N. Hemisphere begins to drift poleward.
- B) An anti-cyclonic eddy in the S. Hemisphere begins to drift poleward.
- C) A cyclonic eddy in the N. Hemisphere begins to drift poleward.
- D) A cyclonic eddy in the S. Hemisphere begins to drift poleward.
- E) A non-rotating parcel eddy in the N. Hemisphere begins to drift poleward.
- F) A non-rotating parcel eddy in the S. Hemisphere begins to drift poleward.



### Vorticity problem



This eddy is located at  $30^\circ\text{N}$  latitude. It is 50 km across.

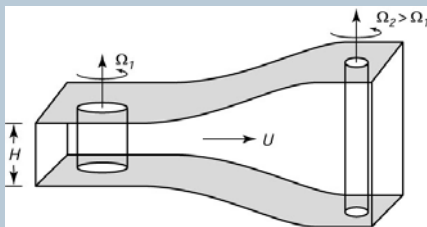
- What is the planetary vorticity?
- What is the absolute vorticity?

If this eddy drifts northward to Greenland, at  $60^\circ\text{N}$ ,

- What is the planetary vorticity?
- What is the absolute vorticity?
- What is its relative vorticity?
- Which way is it spinning?



### Column Stretching



Sketch of the production of relative vorticity by the changes in the height of a fluid column. As the vertical fluid column moves from left to right, vertical stretching reduces the moment of inertia of the column, causing it to spin faster. We are not considering the effect of planetary vorticity here yet. (Stewart)



### Potential vorticity

- We must include depth in the vorticity conservation equation

$$\Pi = \frac{f + \zeta}{D}$$

Depth is a positive number....



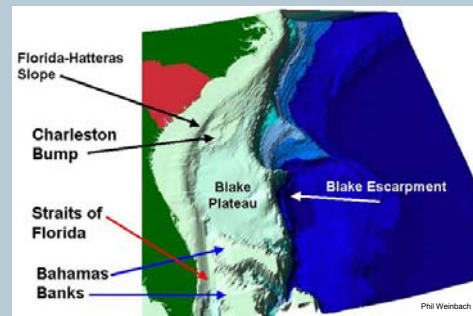
### Potential vorticity questions

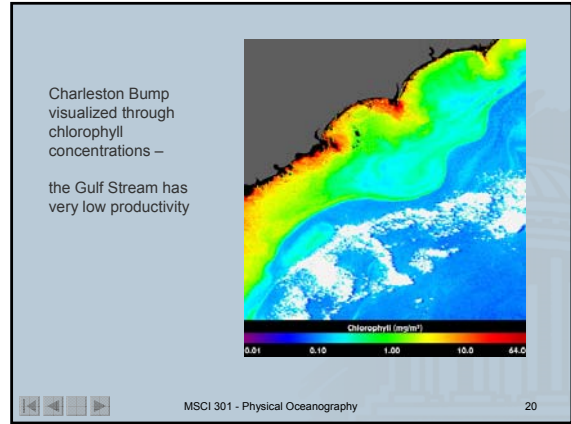
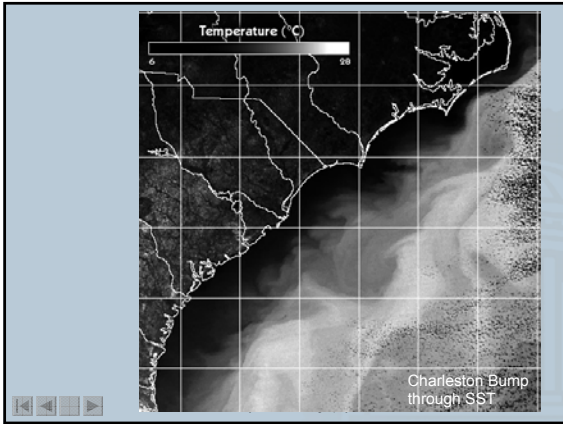
For each of the following cases, determine the possible responses of the water parcel.

- A) An anti-cyclonic eddy in the N. Hemisphere drifts over a seamount.
- B) An anti-cyclonic eddy in the S. Hemisphere drifts over a trench.
- C) A cyclonic eddy in the N. Hemisphere drifts over a trench.
- D) A cyclonic eddy in the S. Hemisphere drifts over a seamount.
- E) A non-rotating parcel eddy in the N. Hemisphere drifts over a seamount.
- F) A non-rotating parcel eddy in the S. Hemisphere drifts over a seamount.



### Real world example





### Revisiting the problem

You find that the eddy you tracked to Greenland is now actually in 4 km deep water, while the water was 2 km deep off of South Carolina.

What was the potential vorticity at 30°N?

What is the potential vorticity at 60°N?

What is the new planetary vorticity?

What is the new relative vorticity?

So what is the eddy doing now?

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### Next time:

How Conservation of Vorticity induces westward intensification in basin circulation

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